Numerical simulation of flow past a square obstacle with Lattice Boltzmann Method

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1. Introduction

Numerical technique Lattice Boltzmann Method (LBM) is relatively new approach based on gas kinetic theory in meso-scale. Macroscopic variables such as pressure and velocity can be determined by momentum of so called particle distribution functions which are described by Boltzmann transport equation. LBM has been extensively studied in last several decades and witnessed as capable like conventional methods for various fluid problems. LBM has its own procedure to solve fluid problems and aforementioned Boltzmann equation (LBE) used to solve fluid flow instead of Navier-Stokes equation (NSE) [1]. To reveal LBM is liable to apply fluid flow, derivation of macroscopic NSE from LBE is made under the Chapman-Enskog expansion which is a multiscale analysis by Chapman and Enskog [2]. LBM is a well-established alternative when it comes to simulate various types of fluid flows including turbulence, multiphase, and porous media flows across complicated geometries [3]. In civil engineering field LBM has been successfully solved free surface flow, groundwater flow and tsunami with shallow water equation. Main advantages of LBM being useful and attractive method in fluid dynamics over the classical approach to solving NSE are its remarkable conceptual simplicity, ease of implementation, predestination to massive parallel computing and straightforward implementation of geometry. In this paper, simplest form of LBM, namely LBM with Bhatnagar-Gross-Krook (BGK) collision operator, is incorporated with turbulent theory and presented for flow around simple square obstacle.

Flow around bluff bodies has been studied for a long time and still an attraction. Most of these studies were concerned with the circular cylinder case under free flow condition in both of traditional and LBM [4]. Flow past rectangular and different shaped bodies were studied extensively but less than circular shape. Practical application of flow past bluff bodies can be found in many fields such as bridge and tall building in civil engineering. Also those well studied problems are considered benchmark to validate code or proposed models in fluid dynamics. In this paper flow past square obstacle is presented by LBM at different Reynolds number of 100 to 3.82x10^4.

2. Extended Lattice Boltzmann method with BGK collision operator

Nature of standard LBM with BGK is devoted to solve incompressible flow under condition of low Mach and Reynolds number. Particularly in high Reynolds number flow, instability of simulation is caused by value of relaxation time which relevant to solving fluid viscosity. To overcome this difficulty, conventional turbulence models such as Large Eddy simulation (LES), Reynolds Averaged Navier-Stokes (RANS) and Direct Numerical simulation (DNS) are incorporating to LBE get extension. To use LES in LBM, one approach is to use Sub Grid Scale (SGS) model in lattice Boltzmann equation [2]. So main equation with BGK collision operator to be solved in LBM framework is given as following discretized form [5]:

\[
f_i(x + c_i \Delta t, t + \Delta t) - f_i(x, t) = -\frac{f_i(x, t) - f_i^{eq}(x, t)}{\tau},
\]

Where \( f_i(x, t) \) and \( f_i^{eq}(x, t) \) are particle and equilibrium distribution functions at position \( x \) and time \( t \), \( c_i \) is the particle velocity along the \( i \)th direction, and \( \tau \left( = 6v + \frac{1}{2} \right) \) is relaxation time that controls the rate of approach to equilibrium. For simulate two dimensional problems, so called D2Q9 (two dimensional nine velocity square lattice) arrangement of lattice with nine discrete velocities \( c_i \) is commonly used. The discrete velocity set is written as

\[
c_i = \begin{cases} c \left( \cos \left( \frac{(i-1)\pi}{4} \right), \sin \left( \frac{(i-1)\pi}{4} \right) \right), & i = 0 \\ \sqrt{2}c \left( \cos \left( \frac{(i-1)\pi}{4} \right), \sin \left( \frac{(i-1)\pi}{4} \right) \right), & i = 1 + 4 \\ \sqrt{2}c \left( \cos \left( \frac{(i-1)\pi}{4} \right), \sin \left( \frac{(i-1)\pi}{4} \right) \right), & i = 5 + 8 \end{cases}
\]

Where \( c = \Delta x/\Delta t \). Equilibrium distribution function works main role of problem specifications and expressed as follow in flow field [1].
\[ f_{\text{eq}} = w_i \rho \left[ 1 + \frac{c_s \cdot \mathbf{u}}{c_s^2} + \frac{(c_s \cdot \mathbf{u})^2}{2c_s^2} - \frac{\mathbf{u} \cdot \mathbf{u}}{2c_s^2} \right], \]

(3)

Where \( w_i \) and \( c_s = \left( \frac{1}{\sqrt{3}} \right) \) are lattice weighting factor for D2Q9 \( w_\theta = \frac{4}{7}, w_i = \frac{1}{7}, \quad i = 1, 2, 3, 4, \) and \( w_i = \frac{1}{36}, \quad i = 5, 6, 7, 8 \) and lattice sound speed, \( \rho \) and \( \mathbf{u} \) are macroscopic density and velocity which can be obtained by momentum of distribution function \( f_i \) as follows:

\[
\rho = \sum_{i=0}^{N} f_i, \quad \rho \mathbf{u} = \sum_{i=0}^{N} c_i f_i, \quad (4)
\]

Where \( N=8 \). To incorporate turbulent model, this paper use introduced approach of SGS in ref. [6]. Modified relaxation time \( \tau_{\text{total}} \) included local eddy viscosity \( \nu_e \) is embedded in equation (1) instead of relaxation time \( \tau \). So total relaxation time is

\[
\tau_{\text{total}} = 3(\nu + C \Delta^2 |\mathbf{S}|) + \frac{1}{2} \tau. \quad (5)
\]

Where \( \nu \) is physical kinematic viscosity and \( C \Delta^2 |\mathbf{S}| (= \nu_e) \) is eddy viscosity term, \( C \) and \( \Delta \) are Smagorinsky constant and mesh size of numeric grid, \( \Delta \) is magnitude of large scale strain rate. In ref. [6], they neglected high order velocity effects to get this magnitude.

\[
|\mathbf{S}| = \sqrt{\nu_e^2 + 18C\Delta^2 \left( \Pi_{xy} \Pi_{yx} \right)^{1/2} - \nu_0}, \quad (6)
\]

Where \( \Pi_{xy} \) can be easily calculated locally in LBM framework as follow,

\[
\Pi_{xy} = \sum_{i=0}^{N} c_{ix} c_{iy} (f_i - f_i^{eq}), \quad (7)
\]

So called the Extended Lattice Boltzmann equation (ExLBM) can be written as;

\[
f_i(x + c_i \Delta t, t + \Delta t) = f_i - \frac{1}{\tau_{\text{total}}} (f_i - f_i^{eq}), \quad (8)
\]

For boundary condition, simple bounce back condition is used to obstacle surface and top and bottom wall of channel while extrapolation method imposed in outlet of domain. Inflow velocity assumed to be uniform.

3. Numerical simulation and code validation

Numerical simulations have been carried out in a domain corresponding to that ref [7] and shown in figure 1. This domain was tested by several authors in turbulence flow with LES, DNS and even experiments. In current study, obstacle side length \( D \) is chosen to be 20 nodes, so Reynolds number and Strouhal number are expressed as;

\[
Re = \frac{3 u_o D}{\tau - 0.5}, \quad St = \frac{f D}{u_o}, \quad (9)
\]

Where \( u_o \) and \( f \) are inflow velocity and vortex shedding frequency, respectively. For bluff bodies, important parameters are drag, lift and pressure coefficients. They are estimated with \( \text{rms and mean} \) value.

\[
C_d = \frac{2 |F_p|}{\rho u_o^2 D}, \quad C_l = \frac{2 F_l}{\rho u_o^2 D}, \quad C_p = \frac{2 (p - p_o)}{\rho u_o^2 D}, \quad (10)
\]

Where \( p(= \rho u_o^2) \) and \( F \) are force and forces acting on obstacle surface, \( p_o \) is reference pressure. Forces on single node can be calculated by momentum exchange with surrounding all possible fluid nodes as follow [8];

\[
F(x, t) = c_i (f_i(x, t) - f_i - (x + c_i \Delta t, t)), \quad (11)
\]

Where \( -i \) is opposite direction of \( i \). Before to simulate high Reynolds number flow, we must validate our result with several ref at \( \text{Re}=100 \) and compared (see figure 2). Standard LBM (SLBM) and Extended LBM (ExLBM) in same conditions also should be compared. In ref [9], they use different domain and periodic inflow condition. Approaching velocity is coincided in all simulation, however past flow presents different behavior which can be impacted by domain distinction and inflow condition. Flow profiles of SLBM and ExLBM are should be same at certain time according to its concept. But in same simulation time and condition, they present different profile which may due to evaluation of eddy viscosity in every lattice in ExLBM. It should be noted that flow pattern and its magnitude were same in SLBM and ExLBM at different time. In ref [9], drag coefficient and Strouhal number were estimated as 1.35 and 0.14, respectively, while in our calculation they are estimated as 1.27 and 0.15, respectively. This nearby result shows that our code works properly in laminar flow. For flow past bluff bodies, Reynolds number higher than 300 are considered in turbulent flow. To validate our code works proper in turbulent regime, we compared pressure coefficient with result of ref [10] shown in figure 4.
In our simulation pressure coefficient drop down more than result of ref [10] in two corner of front face of obstacle. Except two corners, other point’s values are agreeable with references at Reynolds number of 22000. It is notable that combination of chosen value for viscosity and inflow velocity imposes result and numerical oscillation. Streamlines around obstacle is compared with ref [10] and that was also in good agreement. Drag, lift coefficients and Strouhal number are determined as $C_{d}^{rms} = 3.06$, $C_{l}^{rms} = 2.28e-02$, and St=0.126 in current study while those parameters in ref [10] was $C_{d}^{rms} = 2.1$ and St=0.134 at Re=22000. In ref [10], recirculation eddies observed in upper and lower part of obstacle also can be found with finer grid condition in our simulation. Framework of ExLBM use to stress tensor evaluated by Smagorinsky model, so it is obvious to compare vertical velocity of observation points nearest wake of obstacle (see fig 1) to ref results obtained from conventional method [11] incorporated with Smagorinsky model in figure 6.

With same geometry (see fig 1), conventional high order computational framework, called PHEONICS, is used to solve 2D&3D LES simulation with different approach for sub grid Reynolds stress in ref [7]. From the comparison in fig 6, both results of present and ref [7] used to Smagorinsky model are met each other very well while LES with different model has higher amplitude of velocity profile than them. Important parameters such as coefficients and Strouhal number are compared with different results in table 1 at Re=21400 because parameters are suitable to compare and determined from direct effect of distribution functions in LBM framework and averaged over simulation time. In high Reynolds number flow, two case have been compared and in good agreement with references. Currently, up to Re=2.2x104 flow study on square obstacle have not yet been simulated with LBM among survey of previous studies on internet. In this study, flow past square obstacle has studied until Re of 3.82x105 and some of their parameters are put into lower part of table 1.
Figure 6. Vertical velocity plotted on observation point behind of obstacle in different models at Re=21400.

Table 1. Time averaged square cylinder data. Some labels used are the same as in ref. [11]

<table>
<thead>
<tr>
<th>Ref.</th>
<th>Label</th>
<th>$C_l^{rms}$</th>
<th>$C_d^{rms}$</th>
<th>$C_v^{mean}$</th>
<th>$St$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pr.study</td>
<td>ExLBE</td>
<td>0.022</td>
<td>3.081762</td>
<td>0.181</td>
<td>0.159</td>
</tr>
<tr>
<td>[11]</td>
<td>SGS</td>
<td>0.03</td>
<td>2.01</td>
<td>-</td>
<td>0.139</td>
</tr>
<tr>
<td>[12]</td>
<td>ST2</td>
<td>-</td>
<td>2.72</td>
<td>-</td>
<td>0.16</td>
</tr>
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</table>

Parameters at different Reynolds number in present study

<table>
<thead>
<tr>
<th>Re</th>
<th>$C_l^{rms}$</th>
<th>$C_d^{rms}$</th>
<th>$C_v^{mean}$</th>
<th>$St$</th>
</tr>
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<td>0.1</td>
</tr>
</tbody>
</table>

4. Conclusion

In this study, flow past square cylinder has been studied with LBM incorporated with Smagorinsky subgrid scale model at wide range of Reynolds number ($10^2$-$3.82\times10^5$). Numerous results from conventional methods like FD and FVM are compared with present study on corresponding condition with in good agreement which imply that our LBM code works properly and has reasonable accuracy. To be stable simulation over long simulation time, control parameters such as input velocity and fluid viscosity or relaxation time should be chosen in a certain combination. In case of channel flow with square obstacle, decrease of input velocity was good combination with increase of relaxation parameter. Our code to simulate flow past bluff bodies can carry out up to $\text{Re} = 5\times10^5$ with some oscillation on vicinity of obstacle edge. In large scale problem with ExLBM would have computational cost which can be managed by exploit of parallel computation. It should be noted that results in paper are shown in dimensionless form.

Standard LBM has not only limited to apply as shown above turbulent flow, but also restricted to non homogenies and multicomponent flows which are challenged and branched to different type LBM models such as Two Fluid and Phase function model etc. Our future work related to both restrictions. With LBM method we have been striving to solve crucial hydraulics problem such as density current in estuary, sediment transport or bed evaluation and ice formation process in continental rivers.

References